

8

[This question paper contains 4 printed pages]

Your Roll No. :2019.....

Sl. No. of Q. Paper : **7461** **J**

Unique Paper Code : 32351101 - OC

Name of the Course : **B.Sc.(Hons.)
Mathematics**

Name of the Paper : Calculus

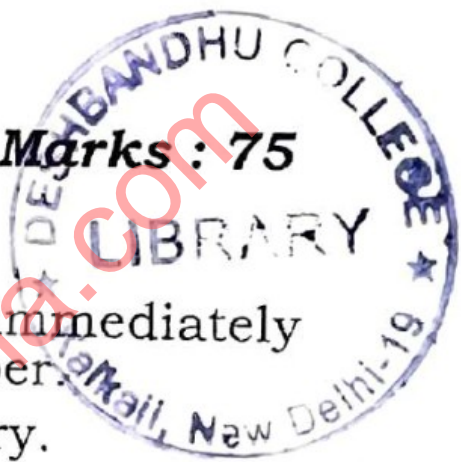
Semester : I

Time : 3 Hours

Maximum Marks : 75

Instructions for Candidates :

- Write your Roll No. on the top immediately on receipt of this question paper.
- All** the sections are compulsory.
- All questions carry equal marks.
- Use of non-programmable scientific calculator is allowed.



Section-I

Note : Attempt any **four** questions from this **Section**.

1. If $y = \sin^{-1} x$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$.

2. Sketch the graph of $f(x) = \frac{1}{x+1} + \frac{1}{x-1}$ by finding intervals of increase and decrease, critical points, points of relative maxima and minima, concavity of the graph and inflection points.

P.T.O.

3. Evaluate analytically following problem :

$$\lim_{x \rightarrow \infty} \left(x \sin^{-1} \left(\frac{1}{x} \right) \right)^x$$

4. Suppose a manufacturer estimates that, when the market price of a certain product is p , the number

of units sold will be $= -6 \ln \left(\frac{p}{40} \right)$. It is also

estimated that the cost of producing these x

units will be $C(x) = 4xe^{\frac{x}{6}} + 30$.

- (a) Find the average cost, the marginal cost, and the marginal revenue for this production process.
- (b) What level of production x corresponds to maximum profit?
5. Sketch the graph of the curve in polar coordinates $r = 4 - 4 \cos \theta$.

Section - II

Note : Attempt any **four** questions from this **Section.**

6. Find the reduction formula for $\int x^n e^x dx$ and

hence evaluate $= \int_0^1 x e^{-\sqrt{x}} dx$.

7. Find the volume of the solid generated when the region enclosed by the curves $y = \sqrt{25 - x^2}$ and $y = 3$, is revolved about x -axis.

8. Use cylindrical shells to find the volume of the solid generated when the region enclosed by the curve $y = \frac{1}{x^3}$, $x = 1$, $x = 2$, $y = 0$ is revolved about the line $x = -1$.
9. Find the exact arc length of the curve $y = \frac{x^6 + 8}{16x^2}$ from $x = 2$ to $x = 3$.
10. Find the area of the surface generated by revolving the curve $x = \sqrt{9 - y^2}$, $-2 \leq y \leq 2$, about y -axis.

Section- III

Note : Attempt any **three** questions from this **Section.**

11. State the reflection properties of the conic sections : parabolas, ellipses and hyperbolas with diagram.
12. Find an equation for the parabola that has its vertex at $(1, 2)$ and its focus at $(4, 2)$.
13. Describe the graph of the equation $9x^2 + 4y^2 + 18x - 24y + 9 = 0$ with rough sketch label the foci, vertices and the ends of minor axis.
14. Trace the conic $x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0$ by rotating the coordinate axes to remove the xy term.

Section - IV

Note : Attempt any **four** questions from this **Section**.

- 15.** Find the position vector and velocity vector if acceleration vector with initial conditions are given as $A(t) = (\cos t)\hat{i} - (t \sin t)\hat{k}$; $R(0) = \hat{i} - 2\hat{j} + \hat{k}$;
 $V(0) = 2\hat{i} + \hat{k}$.
- 16.** A boy standing at the edge of a cliff throws a ball upward at a 30° angle with an initial speed of 64 ft/s. Suppose that when the ball leaves the boy's hand, it is 48 ft above the ground at the base of the cliff.
- What are the time of flight of the ball and its range ?
 - What are the velocity of the ball and its speed at impact ?
 - What is the highest point reached by the ball during its flight ?
- 17.** Find the tangential and normal components of acceleration of an object that moves with position vector $R(t) = (\sin t)\hat{i} + (\cos t)\hat{j} + (\sin t)\hat{k}$.
- 18.** An object moves along the curve in the plane described in polar form $r = 3 + 2\sin t$; $\theta = t$.
 Find its velocity and acceleration in terms of unit polars U_r and U_θ .
- 19.** Find the curvature and radius of curvature at the stated point for a curve
 $x = e^t \cos t, \quad y = e^t \sin t, \quad z = et \quad t = 0$